EFFECTS OF INSTRUCTION USING POLYA'S PROBLEM-SOLVING MODEL ON MATHEMATICAL ACADEMIC ACHIEVEMENT AND ANALYZING ABILITY OF THE FOURTH GRADE STUDENTS, BHUTAN

BY

YESHI DEMA

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The study was one group experimental design. It was carried out to find out the effects of Polya's Problem-Solving Model on Mathematical academic achievement and analyzing ability of the fourth grade students of Trongsa Primary School, Trongsa. Purposive sampling was adopted to select the sample for the study. Pretest, posttest and time series assessment rubrics were the instrument used to collect data and the data was analyzed using mean and t-test with significant p-value 0.05.

The result of the achievement test scores revealed that the mean scores of the pretest was 6.66 and it increased to 13.78 in the posttest. The conclusion drawn was that the Polya's Problem-Solving Model improved students' academic achievement.

The result of the time series record which was maintained to find out the analyzing ability of the students showed that the total mean scores of all the steps of Polya's Problem-Solving Model were: Step1.Understamd the problem (15.3), Step 2. Devise a plan (15.6), Step 3. Carry out (13.8) and Step 4. Look back (11.9). The result confirmed that the students analyzing ability was increased with the help of Polya's Problem-Solving Model.

Student's Signature.......................... Thesis Advisor's Signature......................

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LIST OF ABBREVIATIONS

DCRD : Department of Curriculum Research and Development

NCTM : National Council of Teachers of Mathematics

IOC : Index of Item Objective Congruency

S.R.A.P : Special Recruitment and Admission Program

MOE : Ministry of Education

CAPSD : Curriculum and Professional Support Division

BCSEA : Bhutan Council for School Examination and Assessment

CHAPTER 1

INTRODUCTION

1.1 Background of the study

"Learning mathematics is considered to be important and essential in every aspect of life, but learning mathematics is still a nightmare for many children in the country" (Curriculum and Professional Support Division (CAPSD), 1996). The primary goal of mathematics teaching and learning is to develop the ability to solve a variety of complex mathematics problems. In solving any mathematics problem, one must possess analyzing ability to solve it in a well-structured and in a proper sequence. However, by looking at the process adapted by the students in solving mathematics problem it seems that there is a lack of analyzing skills. Many students feel that learning mathematic is difficult as they do not understand why they learn mathematics and do not see the real world connections. It can be the teaching technique used by the teacher that does not meet the needs of the students. The teachers normally teach what is being prescribed in the text by the ministry regarding the methods used in problem solving. Very limited space is provided to the students to explore and come up with their own creative ways. Most problems were solved by the students by relying on the key words without fully understanding the procedure involved. In other words, when a problem is given to the students, they simply look at the numeral and directly jump to the solving part with what method has been learnt recently without actually trying to find out what the question is asking them to look for. As a result when students confront a real problem they couldn't follow the rote procedure. In mathematics, understanding of the procedure and transferring the understanding to a new situation is crucial.

The fact that Bhutanese mathematics curriculum adopted the standards of National Council of Teachers of Mathematics (NCTM) indicates that mathematics students lacked problem solving skills which is in other word means analyzing skills. One of the most important standards of NCTM was to develop problem solving skills in mathematics. NCTM stated that "solving problems are not only a goal of learning mathematics but also a major means of doing so" (p.52). Therefore it should not be considered as an isolated part of the mathematics program. Problem-solving process will have an imperative influence on teaching and learning, and eventually on students' ability in analyzing the problem. Problem solving is an important component of mathematics education, because it mainly encompasses skills and functions which are important part to analyze different circumstances; analyzing the information in the problem statement, analyzing the best way to solve it using their own questions that guide them to everyday life (NCTM, 1980) by which students can "perform effectively when situations are unpredictable and task demands challenge" (Resnick, 1987:18).

Researchers like Kousar Perveen, Erin McCarthy Bowman, Raji Ayobami Akeem, Riasat Ali and Tzu-Hua Huang, et al, who had applied problem-solving models developed by the first mathematician named George Polya in 1957 in their studies in diverse schools, in different subject areas and it has shown that students performed better after the treatment. Therefore, having seen the importance of good problem solving skills and analyzing skills to be built in students, the present study also intended to apply Polya's model in teaching mathematics in grade four classes in Bhutan. His Problem-Solving Model allows students to understand the problem by identifying known and unknown information in the problem, make a plan using the available information, proceed with the plan and look back on their work done and construct new knowledge. The model acquaints them to solve any problem in a sequential manner, thus, developing positive attitude towards learning mathematics. It is important to note that in mathematics problem solving, the critical point is not reaching to a solution but trying to "figure out a way to work it" (Henderson and Pingry, 1953:248).

1.2 Research objectives

1.2.1 To find out academic achievement in mathematics after using the Polya's Problem-Solving Model.

1.2.2 To find out the analyzing ability in mathematics problem solving after the use of Polya's Problem-Solving Model.

1.3 Research Questions

1.3.1 Does Polya's Problem-Solving Model have effect on Mathematical academic achievement of the fourth grade students?

1.3.2 Does Polya's Problem-Solving Model have effect on the analyzing ability of the fourth grade students?

1.4 Research Hypothesis

1.4.1 Students will exhibit higher learning achievement after using Polya's Problem- Solving Model.

1.4.2 Students will be able to apply analyzing skills with the help of Polya's Problem-Solving Model.

1.5 Scope of the study

1.5.1 Location of the study

The research was conducted in Trongsa Primary School under Trongsa Dzongkhag, Bhutan. The school had classes from pre-primary to six. The research was carried out with students of grade four.

1.5.2 Population, sample and subject

The total population comprised of 347 students of grade four of Trongsa Dzongkhag (District). Sample of the study were the students studying in grade in Trongsa Primary School which is a government school. The researcher used purposive sampling to select one section (section A) as the subject for the experiment. The sample of the study consisted of three section as shown below:

Table 1.1 Sample of the study

Class	No. of students
IV A	32
IV B	32
IV C	33
Total	97

1.5.3 Content

This study was carried out in mathematics subject of grade four developed by Department of Curriculum and Research Development. The following topic from unit Two and Four: Chapter 2- "Division" was covered during the treatment.

Table 1.2 Topics (Content taught)

Units	Chapter Two	Topics
Unit 2- Multiplication and Division Facts	Division	Division as sharing
Unit 4- Multiplication and Division with greater numbers		Division as grouping
		Multiplication and Division fact family
		Division by tens and hundreds
		Estimating quotient

1.5.4 Time

The experiment was carried out for a month. Five lesson plans of 100 minutes each was employed. Two classes of 50 minutes each in a week was taken up for the treatment.

1.5.5 Variables

The two types of variable used in the study are as follows:

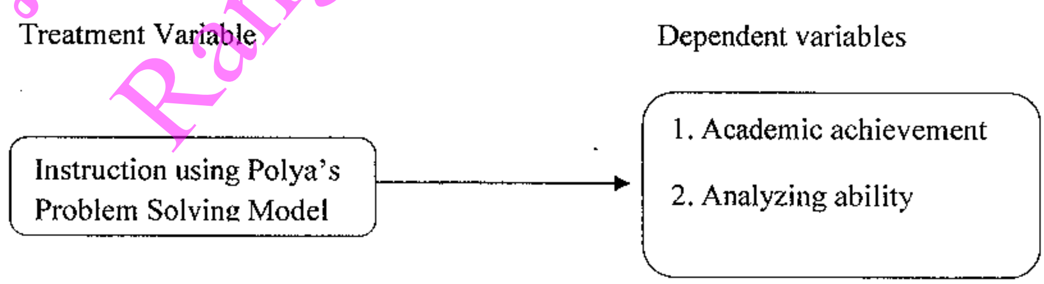

Figure 1.1 Variables

1.6 Benefits of the study

1.6.1 The study would acquaint the students with the problem-solving steps that will enhance their thinking skills in mathematics problem solving.

1.6.2 The findings of the study would be helpful for mathematic teachers in Bhutan to improve their teaching strategy.

1.6.3 This study will serve as reference for future researcher to carry out studies on similar fields.

1.7 Operational definition

Polya's Problem-Solving Model

In this study it refers to the problem solving procedure developed by George Polya in 1957. It consists of four steps in solving mathematics problems1. Understand the problem, 2. Devise a plan, 3. Carry out and 4. Look back.

Analyzing ability

It refers to the thinking skills applied by the students using Polya's Problem-Solving Model to solve mathematics problems.

Mathematics academic achievement

It refers to the marks obtained by the students after using Polya's Problem-Solving Model in mathematics

Trongsa Primary School

It refers to one of the Government School under Trongsa district.

CHAPTER 2

REVIEW OF LITERATURE

In this chapter the literature provides the theoretical background to understand the concept of mathematics problem solving, need for problem solving skills, difficulties in mathematics problem solving, types of problem solving models and related researches.

2.1 Mathematics curriculum of Bhutan

Bhutanese mathematics curriculum was revised in 2008. Before, the mathematics curriculum was based on the Indian curriculum but now it adapted the Canadian system which is more learner-centered. Full cycle of new mathematics curriculum implementation (Pre-primary to grade ten) would be completed by 2014. In Bhutanese context, mathematics learning is more of pattern exploration and relationship of quantities, pace and time. The overall objective of Mathematics Curriculum Reform of Bhutan is to:

a. Make Mathematics education in school relevant, both contextually and conceptually to the learners and the Bhutanese milieu at all levels of school education.

b. Make the teaching and learning of mathematics meaningful so that the learners value the subject.

c. Bring the standard of mathematics curriculum and instruction in Bhutanese school at par with international standards (CAPSD, 2005)

The new mathematics curriculum of Bhutan closely resembles New Brunswick's curriculum, Canada, which explicitly follows principles and standards established by the National Council of Teachers of Mathematics (NCTM). The principles and standards of mathematics problem solving states that students should be able to:

1. Built new mathematical knowledge through problem solving,
2. Solve problem that arises in mathematics and other context,
3. Apply and adapt a variety of appropriate strategies to solve problem, and
4. Monitor and reflect on the process of mathematical problems.

2.2 Problem solving in mathematics

2.2.1 Definition of problem

A problem, according to George Polya (1945) the founder of the problem-solving model and Schoenfeld (1992) defines a problem as situation or a process that an individual or a group of individuals confronts and requires a resolution. It may not necessarily have an apparent or obvious means to obtaining solution. Grouws (1996) defined 'a problem' as "a situation where something is to be found or shown and the way to find or show it, is not immediately obvious" (p. 72). It means that an individual confront with unknown situation and has no distinct means to overcome the situation.

Kantowski (1980) stated that "a problem is a situation for which the individual who confronts it, has no algorithm that will guarantee a solution; the individual's relevant knowledge must be put together in a new way to solve the problem" (p. 195). Jeremy Kilpatrick (1985) defines 'problem' as a situation in which a set target is to be accomplished and a direct means to achieve the set target is blocked. In a similar way, Hoosain (2003) claims that a problem occurs when one is faced with a "given state" and one wants to attain a "goal state". Farooq (1980) pointed out that a 'problem' usually indicates a challenge, the meeting of which requires study and investigation. Basically it means a situation that requires an individual to think, to put effort in finding out the solution and for which the individual does not have any immediate ways of finding the solution.

2.2.2 Definition of problem solving

Literally, problem solving in mathematics is the process to find the solution to a problem when the method is not known to a problem solver. The problem solver has to use strategic skills to select the appropriate techniques for the solution. Polya (1945) defines problem-solving as a process used to solve a problem that does not have an obvious solution. Skinner (1984) states that 'problem-solving' is a framework or a pattern, in which, creative thinking and learning takes place. It is a process of overcoming difficulties that appear to interfere with the attainment of a goal. Bay (2000) explains teaching about problem-solving is the teaching of strategies, or heuristics, in order to solve problems. Problem-solving is a tool used effectively in a wide variety of problems and not a process of memorizing formulas. It is believed that 'problem-solving' approach help distinguishes learning mathematics from the 'memorize-use-forget' approach. In mathematics, memorizing can only help students to solve problems they have already solved earlier but a real problem solver is a very creative, flexible creator who is able to solve problems that he has never encountered before. Brownell (1942) says that problem solving refers: a) only to perceptual and conceptual tasks, b) the nature of which the subject by reason of original nature, of previous learning, or of organization of the task, is able to understand, but c) for which at the time he knows no direct means of satisfaction. d) The subject experiences perplexity in the problem situation, but he does not experience utter confusion. Problem solving becomes the process by which the subject extricates himself from his problem (p. 416).

According to Mayer and Wittrock (2006) problem solving is "cognitive processing, directed at achieving a goal when no solution method is obvious to the problem solver" (p. 287). This definition consists of four parts:

1. Problem solving is cognitive, that is, problem solving occurs within the problem solver's cognitive system and can only be inferred from the problem solver's behavior,

2. Problem solving is a process, that is, problem solving involves applying cognitive processes to cognitive representations in the problem solver's cognitive system,

3. Problem solving is directed, that is, problem solving is guided by the problem solver's goals, and

4. Problem solving is personal, that is, problem solving depends on the knowledge and skill of the problem solver.

National Councilor of Teacher of Mathematics (NCTM) defines the term "problem solving" as mathematical tasks that have the potential to provide intellectual challenges for enhancing students' mathematical understanding and development. Research on problem solving emphasizes the role of the teacher in developing students' analyzing and reasoning skills. In teaching any mathematics class at any level, students should be exposed to a variety of problem solving tasks that require them to collate and analyze previous knowledge and yet offer a challenge (Tripathi, 2009). Polya (1980) once stated that teachers cannot impart the experience of mathematical discovery if they themselves have not had that experience. It clearly states that teacher must understand the nature of problem-solving to effectively apply it in teaching mathematics problem-solving to the students.

As we can see that every definition has common feature that is, students encounter a problem to which they do not have the definite way to solve it to get the solution. To get the solution they need to be familiarized with problem-solving process and encouraged them to use their own knowledge to come up with a new strategy that will help to find the solution. As Gagne (1977) stated that problem-solving is a thinking process in which the learner discovers a combination of previously learned rules that he can apply to solve a novel problem and get the solution.

2.2.3 Problem solving as an analyzing skill

Analyzing skill is defined as the ability to visualize, gather information, articulate and make decision. Leung Kung-shing (1986) also defines analyzing skills as student's ability to break the problem, break the figures, and identify critical elements and abstracting structure. This is same as the procedures followed in Polya's Problem Solving Model, in which a problem has to be broken down in individual phases and tackled accordingly. Therefore, following his steps will naturally address the students' ability to analyze mathematical problems. Analytical skills are considered very important in mathematic problem solving and therefore are commonly addressed as problem-solving skills (Witkin, 1981 as cited by Kung-shing).

Problem-solving is the basic skills that the learners must acquire. Since problem-solving is the central focus and considered the prominent component of the mathematics curriculum, it is necessary for the learners to acquire basic problem solving skills and higher order thinking skills. Use of problem-solving skills not only helps students tackle math problems, but also enables them to work their way logically through any problem they may face. It will help students to learn how to find solution systematically and logically. Problem-solving is a tool, a skill, and a process. As a tool, it helps students to solve a problem or achieve a goal. As a skill it can be used repeatedly throughout the life. And, as a process it involves a number of steps. Problem-solving skills help students develop higher order thinking skills such as, visualization, association, abstraction, comprehension, manipulation, reasoning, analysis, synthesis and generalization - each needing to be managed and coordinated (Garafalo and Lester, 1985). It allows them to use knowledge they bring to school and connect with mathematics situation outside the classroom. Problem solving situations in the classroom provides students the opportunities to experience the power and usefulness of mathematics in the world around them and provides a consistent context for learning and applying mathematics. Therefore, it is important that students should be given problem-solving situations in the classroom from the very earliest stages of mathematics learning. Kousar Perveen (2010) in his study noticed that the foundational stone of cognitive growth and skills takes place in the early years of childhood. This was supported by Khanum (2006) who also noticed that students who are lacking in growth of capabilities and skills definitely faced problems in the next class, as they are not equipped with the base they needed. As stated by Janet Bagby (2000), the more time spent learning to solve problems, the more opportunities learners get to practice and the more likely problem solving transfers.

Moreover, while students are solving problems, they experience a wide range of emotions associated with various stages in the solution process and feel themselves as mathematicians (Taplin, 1988). Problem solving requires students to use previously acquired knowledge, skills, and understanding to meet the demands of an unfamilar situation. This means students are engaged in a task for which the solution method is not known in advance. In order to find the solution, students must draw out their knowledge by analyzing each step, this will help them often to develop new mathematical understandings. Teaching mathematics through a problem centered approach gives students more freedom to approach a problem in different ways and try to understand the problem in a better way, make connections between their out-of-school experiences and school mathematics, and being able to communicate their thoughts by working in small groups or as a whole class. Students learn to choose strategies because they make sense, not just by memorizing what to do in a certain situation. Students will learn from many real-life projects, and will develop more confidence as problem solvers and become mathematical risk-takers. It will increase the opportunities to use critical thinking skills like estimating, evaluating, connection, communicating, representing, relationship, recognizing, and making judgment. Polya (1957) believes that the teachers' can help students at every developmental level to expand the scope of their thinking by using differentiated critical thinking questions in problem-solving activities. Doing so will result in development of analyzing skills.

2.2.4 Difficulties in mathematics problem solving

Students' ability to solve problems is always desirable however there are many difficulties that students frequently experience when they try to solve problems.

They do not understand the problem because of the abundance of sentences. Many students have difficulties sorting the important information from the distracting information in a lengthy problem. Because of their poor reading and comprehension skills, lack of motivation and limited experiences in solving mathematical problems, students have difficulties in transforming word mathematical problems into appropriate numeric format and vice versa. They do not know how to start, they lack the confidence in their ability to solve problems, they lack the skills, tools and access to resources that are needed. It is not that they cannot solve the math problem in most cases, it is simply they do not know the plan and strategies for solving the problem.

Schoenfeld (1985) found that students think mathematics problem can be solved by the only method the teacher uses. Such beliefs lead to rote learning. In mathematics problem solving, rote learning can only help students solve problem that they have encountered before. If they confront a new problem they face difficulty in solving the problem because they are not able to analyze the information provided. Meyer (1978) noted that although certain prerequisite mathematical concepts and skills are related to problem solving success but knowledge of these concepts and skills is not sufficient for successful problem solving. According to Mayer and Wittrock (2006) students need to have five kinds of knowledge in order to be successful problem solvers:

1. Facts: knowledge about characteristics of elements or events, such as "there are 100 cents in a dollar"
2. Concepts: knowledge of a categories, principles, or models, such as knowing what place value means in arithmetic or how hot air rises in science;
3. Strategies: knowledge of general methods, such as how to break a problem into parts or how to find a related problem;
4. Procedures: knowledge of specific procedures, such as how to carry out long division or how to change words from singular to plural form; and
5. Beliefs: cognitions about one's problem-solving competence (such as "I am not good in math") or about the nature of problem solving (e.g. "If someone can't solve a problem right away, the person never will be able to solve it").

The finding indicates that in mathematics problem solving facts and concepts are efficient for representing a problem, strategies are needed for planning a solution, procedures are needed for carrying out the plan, and beliefs can influence the process of self-regulating. To acquire all the knowledge mentioned above to solve mathematics problem, one must possess the procedural skill to analyze the relationship.

Mayer (1982) suggested that some of the types of knowledge that may be relevant for a psychological basis for understanding mathematical problem solving are:

1. Linguistic and factual knowledge - concerning how to encode sentences, Schema knowledge - concerning relations among problem types, procedures and,
2. Strategic knowledge - concerning how to approach problems.

Heller and Hungate (1985) cited by (Berinderjeet Kaur 1997) summarized the nature of the knowledge required for solving problems in complex subject-matter domains as:

a) Knowledge for understanding and representing problems,
b) Strategic knowledge which governs the approach problem solvers take to the task,
c) Knowledge of basic concepts and principles, and
d) Repertoires of familiar patterns and known procedures.

When examining the strategies and behaviors of primary-aged problem solvers, Muir et al (2008) identified a range of skills that were needed in order for success to be achieved. These included interpreting information, planning and working methodically, checking results and trying alternative strategies.

Problem solvers characteristics are the most important determinants of problem difficulty. From the findings of Heller and Hungate (1985) and Kilpatrick (1985) as cited by Kaur (1997), it appears that students must possess relevant knowledge and be able to coordinate their use of appropriate skills in order to solve problems efficiently. Problem solving requires knowledge of strategies and techniques that will help them develop plans for a solution (Polya, 1945). Strategic knowledge governs the approach problem solvers take to the task. In other words, the knowledge one possesses, one's disposition and one's experiential background often influence problem solving performance

With problem solving creating such demands the importance of the students' motivation, engagement and encouragement to internalizing a model that can guide them through the problem solving process is very essential. Polya's said that students who have good understanding of mathematics can still have difficulty applying their knowledge in problem solving because they have not internalized a model that can guide them through the process.

2.3 Types of problem-solving model

There are several different problem-solving models proposed by different experts. One of the many is Dewey (1910) whose model is based on a mixture of theory and practice. He in, "How we think (Boston: D.C. Heath)", outlined five steps:

1. Recognizing that a problem exists - an awareness of a difficulty, a sense of frustration, wondering, or doubt
2. Identifying the problem - clarification and definition, including designation of the goal to be sought, as defined by the situation that poses the problem
3. Employing previous experiences, such as relevant information, former solutions, or ideas to formulate hypotheses and problem solving propositions.
4. Testing, successively, hypotheses or possible solutions. If necessary, the problem may be reformulated.
5. Evaluating the solutions and drawing a conclusion based on the evidence. This involves incorporating the successful solution into one's existing understanding and applying it to other instances of the same problem.

Bransford and Stein (1984) outlined the IDEAL model for problem-solving. They describe problem solving as a uniform process of identifying potential problems, defining and representing the problem, exploring possible strategies, acting on those strategies, and looking back and evaluating the effect of those activities.

1. I - Identify the problem,
2. D - Define it,
3. E - Explore possible strategies,
4. A - Act on the strategies,
5. L - Look at the effect of your efforts

Similarly, one frequently used model of problem-solving is the one developed by Gick in 1986.

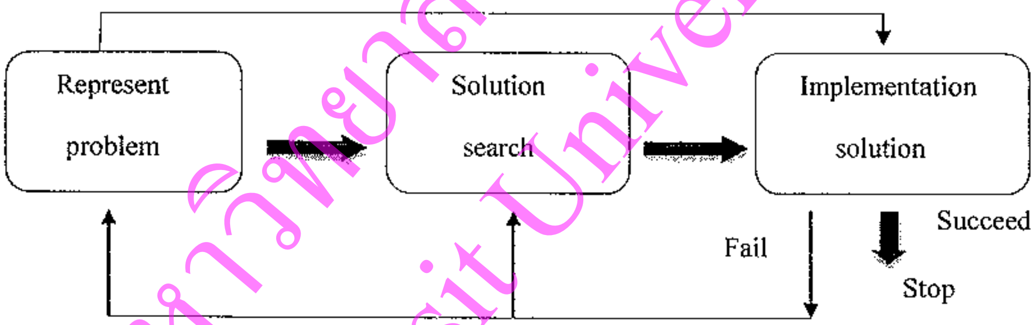

Figure 2.1 Problem solving model

Source: Gick (1986)

His model identifies a basic sequence of three cognitive activities in problem solving:

1. Representing the problem includes calling up the appropriate context knowledge, and identifying the goal and the relevant starting conditions for the problem.

2. Solution search includes refining the goal and developing a plan of action to reach the goal.

3. Implementing the solution includes executing the plans of action and evaluating the result.

Schoenfeld (1985) developed a model based on findings from research by information-processing theorists. His model incorporated Polya's structure and described mathematical problem solving in five episodes: Reading, Analysis, Exploration, Planning/implementation and Verification.

Mayer and Hegarty (1996) examined mathematics problem solving in terms of four components: Translating, Integrating, Planning and Executing. They hypothesized how expert problem solvers use different strategies from novice problem solvers in these four components (it was stated in his study Margaret Wu and Raymond Adams)

2.4 Polya's Problem-Solving Model

Amongst many models, George Polya's Problem-Solving Model is one which is widely used. In his renowned publication, "How to Solve It", Polya (1957) suggested that solving a problem involved: i) Understanding the problem; ii) Device a plan; iii) Carry out the plan; iv) Look back. He described the problem solving process as a linear progression from one phase to the next and advocated that when solving a problem;

First, students have to see clearly what is required. They have to identify known and unknown information provided in the problem. According to Polya, student should not only understand the problem but also desire its solution. He believes that teachers can check the understanding to some extent by asking questions.

Second, students have to see how the various items are connected, how the unknown is linked to the data, in order to obtain the idea of the solution and to make a plan. Polya mentioned that there are many possible ways to solve a problem. The skill at choosing the appropriate strategy is by solving many problems. The materials necessary for solving mathematics problem are certain relevant items of formally acquired knowledge, formerly solved problem and formerly proven theorem.

Third, students carry out the plan. The plan provides the outline however the plans need to be thoroughly examined one after another, patiently. Persistent with the plan that student has chosen, if it continues not to work student can discard it and choose another.

Fourth, students have to look back at the completed solution; we review it and discuss it. The problem is often not completely understood until the problem-solver has tried and failed to arrive at a solution using different strategies (Polya, 1957). He mentioned that much can be gained by taking time to reflect and look back at what you have done, what worked and what didn't. It is a series of going forward and backward among the four stages of the model. This will help to predict what strategies to use to solve future problem. The figure below represents Polya's Problem-Solving Model.

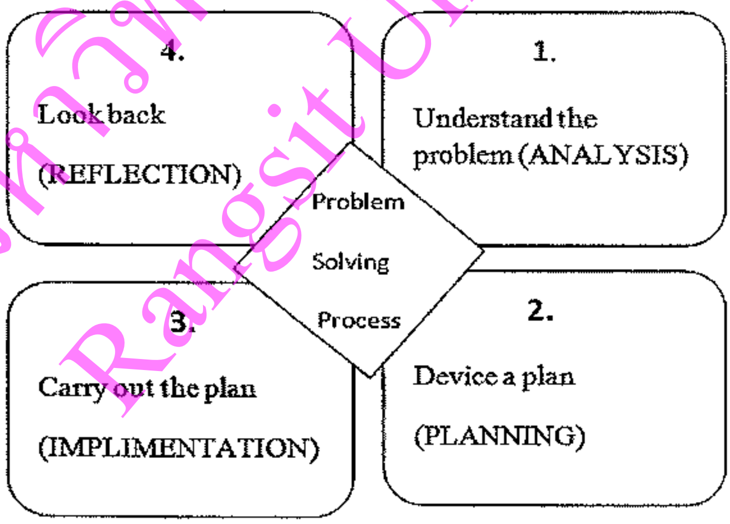

Figure 2.2 Problem-Solving Model of George Polya

Source: http://www.learnlogic.net/polyas-problem-solving-proces, 21 November 2012

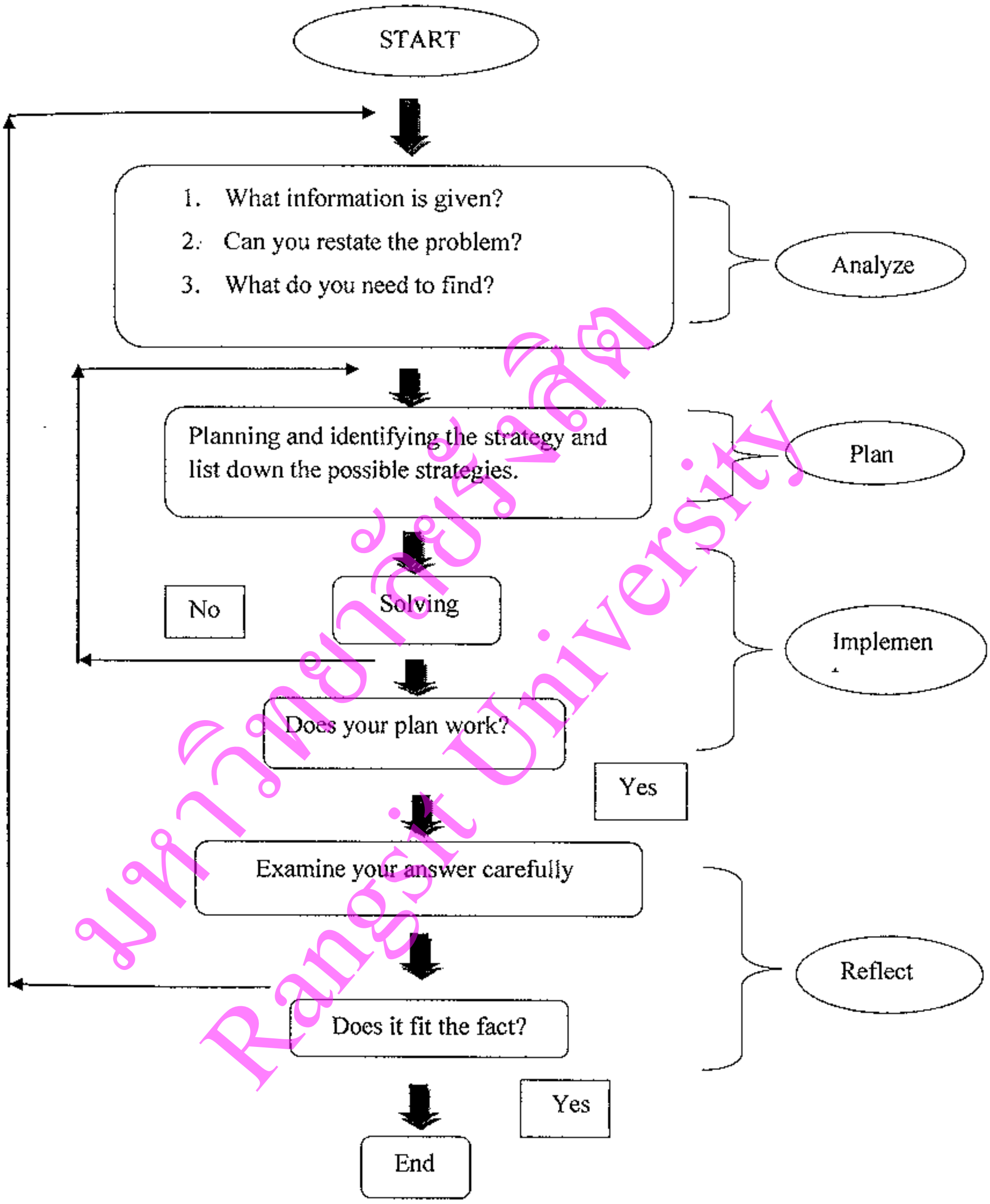

Figure 2.3 Procedure of Polya's Problem-Solving Model

Source: http://jmat.ipgmksm.edu.my/pdf/ps.pdf, 21 November 2012.

2.4.1 Polya's principles

George Polya in his book "How to Solve It", he explains various principle and techniques used in solving problems. He mainly focuses on teaching techniques for solving mathematics problems efficiently. Polya wrote his book as a guide for teachers and students alike on how to solve mathematical problems of all kinds in a systematic way by asking the right questions and looking for the answers in the right places. He highlights many essential ideas and teaching suggestions that can be used in transmitting the information to the students.

Polya says that experiences play a vital role in transmitting the information. He writes "in order to be able to see the student's position, the teacher should think of his own experience, of his difficulties and successes in solving problems." Teachers must be able to look at the problems from the student's point of view. If teachers fail to do so, then the task of conveying the concepts becomes significantly more difficult. He also advises teachers to demonstrate to students the depth of every problem. He says that "good teacher should understand and impress on his students the view that no problem whatever is completely exhausted." A problem is not merely a small link in a chain of problems set up to drill concepts into the heads of mathematical novices. Rather, each and every problem can be used to shed light on a great number of ideas. For this reason, Polya stresses that a good teacher use each problem to demonstrate the depth of mathematics. Polya also stresses the importance of giving students the proper suggestion. He points out that when it comes to aiding a student in solving a problem, the teacher must find a balance between two extremes. On the one hand, the teacher must not explain every single step to the point where no input is necessary on the part of the student. The student must be involved in the solving process. At the same time, the teacher must give enough information so that the student has something to work with and to develop. The teacher cannot be completely removed from the student, and must not leave the student to figure out the whole thing by themselves. Otherwise, the teacher serves no purpose.

Polya says that the suggestions should be "simple and natural." Mathematics can be very difficult, but things are made much easier if basic questions are asked. The teacher should hint to the student by suggesting methods that are not complex and do not isolate the student. Also, Polya makes clear that the suggestions must be general and not particular to the problem at hand. The main benefit is that the student will be able to use the method to other problems they confront. Thus, the teacher conveys more than just a clue to this problem. This general suggestion gives the student a key to a whole host of problems that he may encounter. In addition, by making the suggestion general, the teacher still have the student to work out the particular solution himself.

Polya gave a good example on this idea with relation to a 'connect-the-dots puzzles' The dots are drawn on the page by one person and are used by someone else as an outline to produce a complete picture. The teacher must know how to present the student with dots, that is, the primary principles used in solving a particular problem. The student connects the dots to paint a complete picture. That is, he uses the general advice from the teacher to construct a solution to the problem.

2.4.2 Related learning theories

Learning in defined as the process that brings together cognitive, emotion and environmental influences and experiences to determine how one acquire or make changes in one's knowledge or skills. Learning as process focuses on what happens when learning takes place. Description of what happens constitute of learning theories. Learning theories are conceptual framework that describes how information is absorbed and retained during learning. According to Hill (2002), learning theories have two chief values; one is providing vocabulary and conceptual framework for interpreting the examples of learning that are absorbed. And the other one is in suggesting where to look for solution. The following learning theories support Polya's Problem-Solving Model.

2.4.2.1 Behaviorism

Behaviorism as a theory was primarily developed by B.F.Skinner. He states that changes in behavior are the result of an individual's response to events (stimuli) that occur in the environment. Behaviorists' understanding of learning was based on the cause and effect. Two problem-solving methodologies explain the problem solving process within the framework of behaviorist learning theory. One such method is trial and error. This involves attacking the problem by various methods until a solution is found. Young children solving a jigsaw puzzle exhibit this type of problem-solving behavior. The children try fitting different pieces into the same spot until finally they find the piece that fits. Another method consistent with behaviorist learning is Hull's response hierarchy. This method involves learned responses that are applied to a situation in a hierarchical manner. The hierarchy is based on the response for which habit strength is strongest. Stimuli in a problem situation may evoke several different responses, and responses will be produced, one at a time, in order of strength, until either the problem is solved or the organism exhausts its repertoire of responses.

Polya mentioned that problem is often not completely understood until the problem solver has tried and failed to arrive at solution using different strategies. His problem solving process is a series of going forward and backward among the four steps. Students have to look back at each step and if the first plan fails they have to try out another strategy until they come to solution.

2.4.2.2 Cognitivism

George Polya was a Cognitive psychologist who came up with cognitive heuristics. Literally, heuristics means general problem solving methods. This heuristics comes within the framework of Polya's four steps model. Some of the heuristics applied in the plan include understanding the unknown, simplifying the problem, understanding the nature of the goal state etc. Mayer and Wittrock (2006) distinguished among four major cognitive processes in problem solving:

Representing, in which the problem solver constructs a cognitive representation of the problem; Planning, in which the problem solver devises a plan for solving the problem; Executing, in which the problem solver carries out the plan; and Self-regulating, in which the problem solver evaluates the effectiveness of cognitive processing during problem solving and adjusts accordingly. The same cognitive process occurs in Polya's Problem-Solving Model where students firstly identifies known information to represent the problem, followed by devising a plan to solve the problem, next students carry out the plan, lastly they look back to check their work.

The cognitive view of learning sees knowledge as given and absolute. Many of the information processing models of teaching and learning are based on the cognitive view of learning. Cognitivist focuses learning as a mental operation that take place when information enters through the senses, and undergoes mental manipulation which is stored and finally used. Cognitive theories focus on unobservable processing, storage, and retrieval of information of the brain. According to cognitive learning theories, the individual acts, originates and thinks and this is the important source of problem solving. Piaget's theory of cognitive development proposes that humans cannot be "given" information which they immediately understand and use. Instead, learners must "construct" and "reconstruct" their own knowledge. To construct new knowledge, a child must operate on objects and through this action students gain new knowledge of that object.

George Polya Polya's model expects students to think about strategies, tactics, and patterns available to them to solve problems (Wilson, Fernandez, & Hadaway, 2005 cited by Hill, 2008). His model focuses on how to do mathematics and how to reflect on problems solved. Polya (1957) saw problem solving as more than a process taught to students. He conceived of problem solving as a way of thinking and analyzing. His heuristics provide strategies that help students think, interpret, and reflect on problem situations rather than simply what "to do" when tackling a problem situation. For example: drawing a picture when solving a problem depends on the problem solver's ability to interpret the problem rather than his or her ability to make a drawing. Problem solving becomes the process of mathematically interpreting a situation involving iterative cycles of understanding given information and the need to solve a problem situation. Problem solving involves the problem solver developing an effective way of thinking about a given problem situation. According to Lester and Kehle (2003), "problem-solving is an activity requiring the individual to engage in a variety of cognitive actions, each of which requires some knowledge and skills and some of which are non-routine" (p. 510).

The role of the cognitivist teacher is to support the learner's application of the proper learning strategies, and encourage the learners to actively participate in the learning process. Cognitive leaning teachers view mistakes as unsuccessful efforts to understand order and act upon their environment in ways that make sense to them. Gordon (1994) stated that knowledge is the organization of a set of mental structures and problem-solving processes that the learner manipulates and restructures in response to new information and experience. Teachers should be concerned with the process of learning rather than the end product. For example, the teacher should monitor the way a child play with dough instead of concentrating on a finished shape. In Mathematics it implies that process in more important than the product.

2.4.2.3 Constructivism

Based on the constructivist theory, when individuals deal with the physical world their minds construct through certain mental mechanisms, collections of cognitive structures that enable them to conceptualize, reasoning, and coordinate their engagements. The constructivist view involves two principles: Knowledge is actively constructed by the learner, not submissively received from the environment, and coming to know is a means of adaptation based on and frequently modified by a learner's experience of the real world (Von Glasersfeld, 1987, Schoenfeld, 2002, Jaworski, 2006, Voskoglou, 2007, cited by Michael Gr. Voskoglou, 2008). The theory focuses on the understanding of the information. Polya said that to construct new knowledge students should be guided with questions. This helps students to make connection with their existing knowledge.

Bruner's theoretical framework is also based on the theme that learners construct new ideas or concepts based upon existing knowledge. Learning is an active process. Facets of the process include selection and transformation of information, decision making, generating hypotheses, and making meaning from information and experiences. Bruner's theories emphasize the significance of categorization in learning. To observe is to categorize, to conceptualize is to categorize, to learn is to form categories, to make decisions is to categorize. He presented the point of view that children are active problem-solvers and capable of exploring "difficult subjects". He believed learning and problem solving emerged out of exploration. Constructivism takes place in problem solving situations where learners draw on their own experiences and knowledge to ascertain facts and new truths to be learned (Bruner, 1960).

In mathematics problem solving, constructivist perspective has led to an approach to teaching which emphasizes on children to actively engage in mathematical task and construct their own meaning. Problem solving involves using prior knowledge, experiences, and familiar representations to make sense and attempt to obtain new information about the problem solving situation. When the students fails to directly apply his or her mathematical knowledge to a problem solving situation they must transform or find a new perspective on their knowledge so it can be applied to the problem situation. This process of transforming their knowledge and finding new perspective could lead to the development of new mathematical ideas (Lester & Kehle, 2003). Problem solving enables students to construct and test mathematical theories, solve their own problems, and discuss these theories, leading to an increase in students' mathematical understanding.

Teacher has a vital role to invest in helping students construct new knowledge through solving problem process. The teacher's role is that of a facilitator and requires considerable reflection as the teacher must observe student responses, challenge students thinking and encourage risk taking within a supportive classroom environment. Teacher should encourage students to question themselves and their techniques in the process of problem solving. Doing so helps students develop analyzing skills.

2.4.2.4 Gestalt theory

The Gestalt theory of problem solving, described by Karl Duncker (1945) and Max Wertheimer (1959), holds that problem solving occurs with a flash of insight. Richard Mayer (1995) noted that insight occurs when a problem solver moves from a state of not knowing how to solve a problem to knowing how to solve a problem. During insight, problem solvers devise a way of representing the problem that enables solution. Gestalt psychologists offered several ways of conceptualizing what happens during insight: insight involves building a schema in which all the parts fit together, insight involves suddenly reorganizing the visual information so it fits together to solve the problem, insight involves restating a problem's givens or problem goal in a new way that makes the problem easier to solve, insight involves removing mental blocks, and insight involves finding a problem analog (i.e. a similar problem that the problem solver already knows how to solve). Gestalt theory informs educational programs aimed at teaching students how to represent problems.

2.4.3 Related research

Kousar Perveen (2010) conducted a study in the Government Girls High School Rawalpindi to find the Effect of the Problem-Solving approach on academic achievement of students in mathematics at the secondary level. The sample size consisted of 48 students. The experimental group was taught using a series of lesson plans put together with the help of Shireen (2006) and Polya (1945) guidelines, which include heuristic steps of the problem-solving approach. The result of the finding revealed the effectiveness of the problem-solving approach.

Erin McCarthy Bowman (2010) also conducted a research on evaluating word problem using Polya's Problem-Solving strategies. His study was attempted to determine whether Polya's Problem-Solving plan was an effective strategy for improving middle school ESL (English as second language) students' oral and written communication of their mathematical thoughts when solving word problems. The study demonstrates that Polya's plan helps ESL (English as a second language) students organize their thinking before trying to convey their message and the study also noted that the teaching of Polya's problem-solving strategy improved ESL students' abilities to understand what information is important in a math word problem and what information students still need to find.

Riasat Ali (2010) examined the effect of problem solving method on students' achievement in teaching mathematics at elementary level. Seventy six 8th grade students' of Government Girls Higher Secondary School Ghoriwala, Bannu, Pakistan, were taken as the sample of the study. Pretest and posttest design was used to collect data and it was analyzed using mean, standard deviation and t-test. The finding of the study concluded that there was significant difference between the traditional teaching method and problem solving method. It also revealed that problem solving method enhanced the achievement of students' mathematics.

Halil Eksi (2005) carried out study to investigate the effect of different problem-solving strategies on freshman university students' achievements of quantitative problems in a general chemistry course. In order to identify students' achievements of quantitative problems in chemistry, the Quantitative Problem-solving Achievement Test consisting of 20 multiple-choice items was developed and administered to 150 students as pre-and-post tests. The results of ANCOVA indicated that the students' achievements of quantitative problems in the Polya's Problem-Solving strategy incorporating with cooperative learning approach were better than the students' achievement of quantitative problems in traditional problem-solving and the Polya's problem-solving strategies. Based on the findings, it can be concluded that this approach helps the development of students' problem-solving skills and achievements because using the Polya's problem-solving strategy with cooperative learning may increase verbal interaction among students.

William Arthur Schurter (2001) conducted study on "Comprehensive Monitoring and Polya's heuristics as tools for problem solving". His study comprises of three groups and was taught using different methodologies. The first group was taught using traditional method (controlled), second group was taught using comprehensive monitoring alone (experimental) and the third group was taught with conjunction of Polya's 4-step method (experimental). He used pre-test, post-test, interview and questionnaires to collect the information. The information gathered was analyzed using ANOVA and t-test. The result of the finding stated that students who received increase emphasis on the use of comprehensive monitoring alone and in conjunction with Polya's heuristics both seems to perform better in mathematic problem-solving then those who did not receive either type of instruction.

Victor Uche (1986) conducted a study to investigate the effect of problem solving instructional method on the mathematics achievement. In his research he used pre-test and post-test to collect data. The data was analyzed using SPSS. The study found out that problem-solving method was more effective and beneficial to the students of Special Recruitment and Admission Program (S.R.A.P).

Newfoundland (1980) conducted a study of the effect of teaching heuristics on the ability of 10th grade students to solve novel mathematical problems. A group of ten boys were taught by the use of self-instruction booklets to apply the heuristics of examination of cases and analogy to novel mathematical problems. The data, analyzed by ANOVA, indicates that: 1) students can be taught to apply at least one heuristic to a novel problem; 2) it is better to teach heuristic alone than to combine the instruction with the teaching of mathematical content; and 3) the ability to apply at least one heuristic is independent of the vehicle used to introduce it. The evidence suggests that heuristic-oriented instruction can be an effective mode for teaching mathematical problem-solving.

2.5 Conclusion

Learning how to solve problems in mathematics is knowing what to look for. Math problems often require established procedures and knowing what and when to apply them. To identify procedures, students have to be familiar with the problem situation and be able to collect the appropriate information, identify a strategy or strategies and use the strategy appropriately. Problem solving requires students to think about context of the problem, choose strategies to solve a problem, compute the answer and explain their reasoning. Simple problems within our textbook are important elements to practice with our students. Students should be given opportunities to solve problem as early as kindergarten with basic problem-solving skills. Starting early lays the foundation for later years. Problem solving opportunities should be balanced with real world and abstract situation. Problem-solving is the basic skill needed and it encourages students to use higher order thinking skills. In addition, problem solving is a life skill that will serve students well throughout their life. Research on problem solving emphasizes the role of the teacher in developing students' reasoning skills. Knowledge cannot be transmitted from the teacher to a student in a way that information is copied and pasted into the students head. The student must be involved actively in learning so that he could reach his own insight by individual work.

In teaching any mathematics class at any level, students should be exposed to a variety of problem solving tasks that require them to collate and analyze previous knowledge and yet offer a challenge (Tripathi, 2009). Teacher's role is to choose appropriate teaching forms, methods and encourage independent discovery.

CHAPTER 3

RESEARCH METHODOLOGY

This chapter describes the general procedure that was adopted to collect data for the study. It was one group experimental design. The chapter consists of the research design, research instruments, its validity and reliability and lastly, it describes how data was collected and analyzed.

3.1 Research design

The research type is the experimental in nature and both qualitative and quantitative method were adopted. In the study, academic achievement test and time series design was used to collect data to find out students' academic achievement after incorporating Polya's Problem-Solving Model and to determine the analyzing ability of the student in solving problems. Figure 3.1 illustrates the design

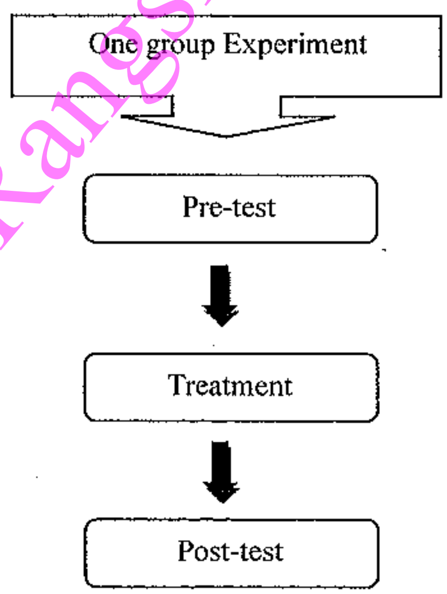

Figure 3.1 Research process

3.2 Sample and Subject

The sample of the study comprised of 97 students of fourth grade studying in Trongsa Primary School, Trongsa, Bhutan and the subjects of the study comprised of 32 students.

3.3 Treatment procedures

Firstly the researcher oriented the subjects on the use of Polya's Problem-Solving Model as they had no prior experiences on its use. This was done in the first session of the treatment period. The researcher explained each step with examples and also conducted short activities to help them get familiarized with the model. The researcher also taught all the methods prescribed in the text used in solving Division problem along with the model because the methods used came under the framework of Polya's Problem-Solving model.

After the orientation the researcher taught the topic 'Division' to the students. The topics were divided into five subtopics as shown in table 1.1. The researcher used varieties of teaching materials which are in line with the constructivist principles to make the learning more interesting, meaningful and to get hands on learning experiences.

3.4 Research instruments

3.4.1 Experimental instruments

Five lesson plans were used in this study. The lessons were prepared from unit 2, Division topic in mathematics and it was taught incorporating Polya's Problem-Solving Model. Each lesson was planned for 100 minutes (block period). The lesson was planned using the four steps of Polya's Problem-Solving Model but following the format developed by Ministry of Education (MOE), Bhutan.

3.4.2 Data collection instruments

3.4.2.1 Achievement test

Pretest and posttest were administered to examine the academic achievement before and after incorporating the Polya's Problem-Solving Model in mathematics. The scores of pretest and posttest of individual subject were compared using t-test to see the level of achievement in mathematics at the end of the treatment.

3.4.2.2 Time series

Time series is simply a sequence of numbers collected at regular intervals over a period of time. Time series in this study was employed to collect information on progress of analyzing ability applied using Polya's Problem-Solving Model in mathematics. To collect the information researcher conducted a short test every end of the lesson and it was assessed using Polya's assessment rubric. The marks obtained by the students were recorded. The test consisted of two questions each and each question carried 16 points (4 points for each step as per the rubric).

3.4.3 Content validity and reliability

Validity of the instrument was examined by the panel of experts from Rangsit University and mathematics teachers from Bhutan. Reliability of the instrument was done using Kuder-Richardson (KR-20). The test coefficient value was 0.77 indicating that the instruments were reliable.

3.5 Data collection procedures

An approval letter from the Director General, Ministry of Education of Bhutan was obtained. Five detailed lesson plans was employed in teaching mathematics to the fourth grade student in the school. Each lesson was planned for 100 minutes incorporating Polya's Problem-Solving model. Pretest and posttest of 20 multiple choice items prepared based on the question format used by the Bhutan Council for School Examination and Assessment (BCSEA) was administered in the beginning of the experiment and at the end of the treatment period respectively. Time series design was used to keep records of students' progression in application of analyzing skills with the use of Polya's Model. For the record a test was conducted every end of the lesson. The test consisted of two questions each of 16 points and it was assessed using Polya's Problem-Solving Model rubric.

3.6 Data analysis

Data analysis was carried out in two areas:

1. Learning achievement in mathematics after using Polya's Problem-Solving model,

The inferential statistics t-test with $P < 0.05$ level of significance was used to analyze the data to compare pre post-test achievement of the students.

2. Analyzing ability of the students in mathematics problem solving

To check the analyzing ability of the students, the scores obtained by the students from the test conducted every end of the lesson were analyzed and used in comparing the analyzing skills applied firstly within the Polya's four steps, secondly between whole steps of each lesson and lastly between the different ability (High, Average, Low) groups.

CHAPTER 4

RESULTS OF DATA ANALYSIS

The motive of this study was to find the effect of Polya's problem-solving model on the academic achievement and the analyzing ability of fourth grade students in one of the primary schools in Trongsa, in mathematics subject. Since the study was one group experiment, only 32 students (17 boys and 15 girls) took part. Five lessons were taught using Polya's problem-solving model from the topic Division. The data was collected from pretest and posttest and was analyzed using t-test with significant value of (0.05). Time series records were maintained from four lessons to check the student's analyzing ability after using the Problem-Solving Model.

The results of data analysis are presented as follows:

1. Analysis of pre-test and post-test result of experimental group
2. Analysis of time series records from four lessons on analyzing ability of experiment group

4.1 Analysis of pretest and posttest results

The pre and post-test (Appendix D) was administered on experimental group to determine the learning achievement before and after using Polya's Problem-Solving Model in teaching mathematics in fourth grade.

Figure 4.1 illustrated the comparison of the means of pretest and posttest

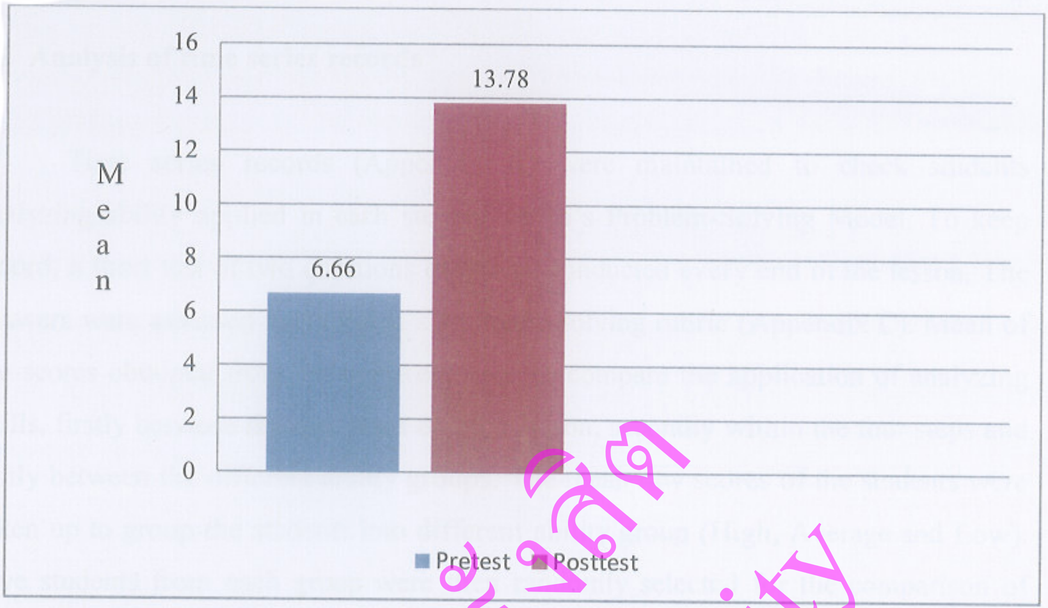

Figure 4.1 Pretest and posttest mean comparison

Table 4.1 shows the comparison of mean scores of pretest and posttest

Table 4.1 Mean and S.D of pretest and posttest result

	N	Mean	S.D	Mean difference	d.f	Sig.(2-tailed)
Pre test	32	6.66	2.44	7.12	31	0.000
Post test	32	13.78	3.02			

The mean of pretest was 6.66 and the standard deviation was 2.44. In the posttest the mean was 13.78 with standard deviation 3.02. The mean increased by 7.12.

From the result it was apparent that there was increase in the mean of the posttest than that of the pretest. The dependent t-test was applied and the 2-tailed significance value 0.000 ($p < 0.05$) at the d.f (31) indicated that the learning achievement of the students were statistically significant after the treatment.

4.2 Analysis of time series records

Time series records (Appendix B) were maintained to check students analyzing ability applied in each step of Polya's Problem-Solving Model. To keep record, a short test of two questions each was conducted every end of the lesson. The answers were assessed using Polya's Problem-Solving rubric (Appendix C). Mean of the scores obtained from the test were used to compare the application of analyzing skills, firstly between the four steps of each lesson, secondly within the four steps and lastly between the different ability groups. The mean raw scores of the students were taken up to group the students into different ability group (High, Average and Low). Five students from each group were then randomly selected for the comparison of their analyzing skills applied in each step.

4.2.1 Comparison of mean between the steps of Polya's Model

The analysis of the time series record was firstly done by comparing the mean scores of individual subject obtained in each lesson (test).The figure below shows the level of analyzing skills applied by the students in all the steps in each lesson (test) of the Problem-Solving Model.

	Lesson 1	Lesson 2	Lesson 3	Lesson 4
Step 1	3.0	3.8	4.0	4.4
Step 2	3.6	3.6	4.1	4.3
Step 3	3.1	3.4	3.7	3.6
Step 4	2.6	2.8	3.2	3.3

Figure 4.2 Level of analyzing ability in all the steps of Polya's Problem-Solving Model

From the table it was clear that with every new lesson, the students demonstrated improvement in application of analyzing skills. The mean of the first step: Understand the problem was 3.0 and it increased to 4.4 by the end of the treatment. Similarly, the mean of step two: Devise a plan and step 4: Look back increased with mean 4.3 and 3.3 respectively. While the mean of third step: Carry out, the mean went down by 0.1. The result also revealed that the students had applied very less analyzing ability in all the steps in the beginning of lesson (lesson 1). Gradually students demonstrated that they applied analyzing skills with every new problem solving situation indicating progression. By looking at the increase in application of analyzing skills it was concluded that the Polya's Problem-Solving Model enhanced student's analyzing ability, ultimately resulting in improvement in mathematics performance.

4.2.2 Comparison of total mean scores within the four steps of Polya's model

The comparison of total mean scores within the steps was done to study the application of analyzing skills by the students in each step. The figure below illustrates the trend of analyzing skills applied in each step.

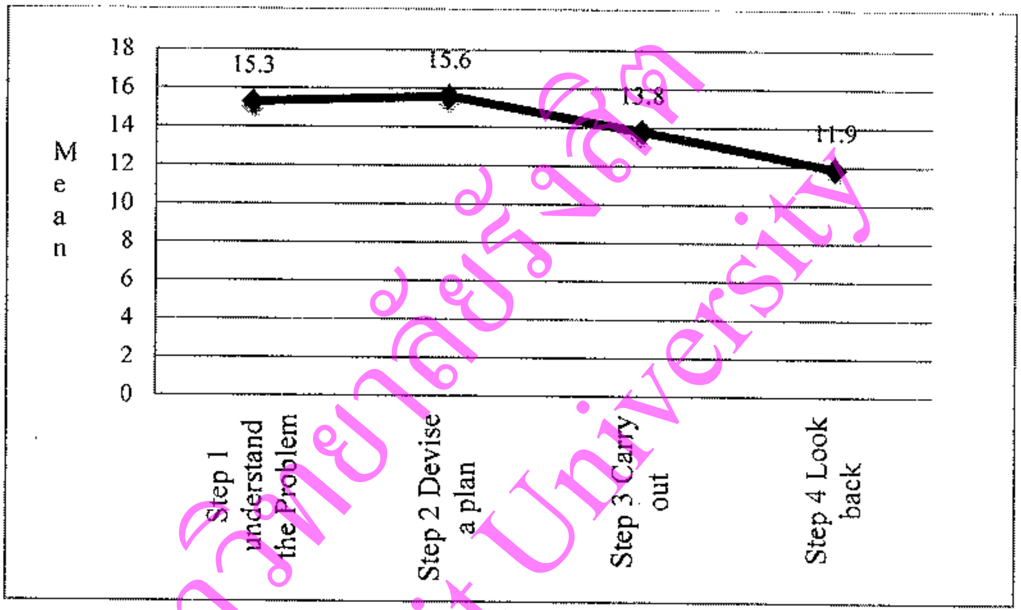

Figure 4.3 Total mean of analyzing ability in each step of the Problem-Solving Model

The figure above indicated the total mean scores of analyzing skills applied by the students in step 1(Understand the problem) was 15.3, step 2(Devise a plan) was 15.6, step 3(Carry out) was 13.8 and the last step 4(Look back) was 11.9. The result revealed that the students performed well in step 2 (Devise a Plan) indicating maximum application of analyzing skills. While in the last step students performed low indicating minimum application of analyzing skills in all the lessons.

4.2.3 Comparison of subtotal means raw scores of different ability groups

Comparison of subtotal mean raw scores between the different ability groups was done to see the analyzing skills applied by each group in each step and also to see if the Polya's Model is applicable for the different learners.

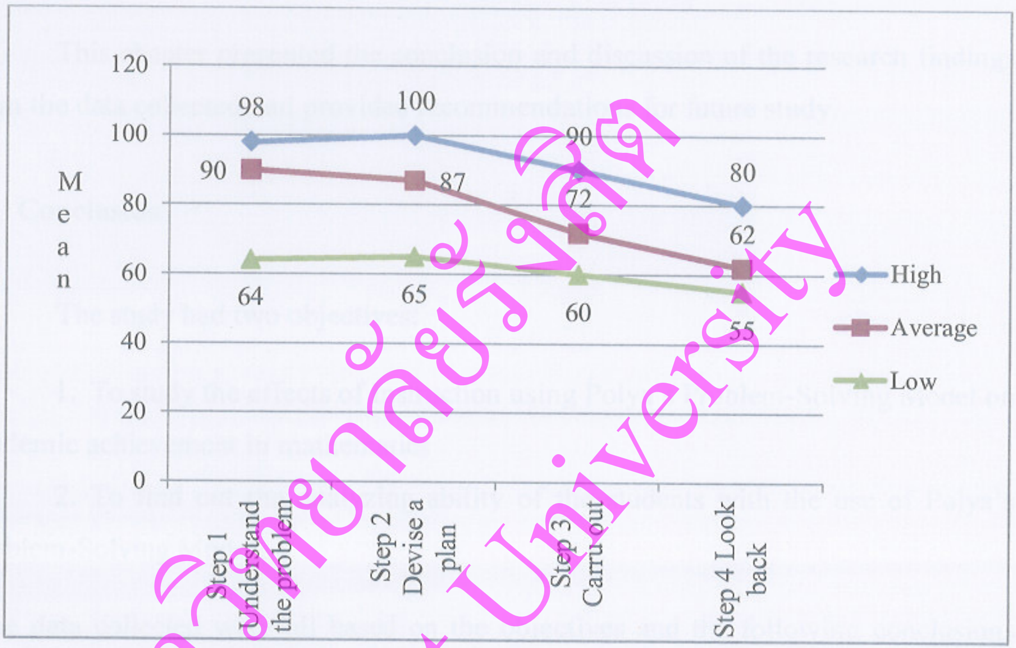

Figure 4.4 Subtotal mean raw scores of analyzing ability in each steps by different groups

The figure above indicated that different ability group faced the same problem when coming to the use of Polya's Problem-Solving Model though their scores obtained differed from each other in all the steps. High achiever and low achiever could perform better in the second step (Devise a plan) and the average group could perform well in the first step (Understand the problem). However the level went down in step 3 and 4. This revealed that all the groups equally had difficulty in implementing the plan and had very limited ideas on how to go about checking the answer and looking back at the first three steps.

CHAPTER 5

CONCLUSION AND DISCUSSION AND RECOMMENDATIONS

This chapter presented the conclusion and discussion of the research findings from the data collected and provided recommendations for future study.

5.1 Conclusion

The study had two objectives:

1. To study the effects of Instruction using Polya's Problem-Solving Model on academic achievement in mathematics
2. To find out the analyzing ability of the students with the use of Polya's Problem-Solving Model.

The data collected were all based on the objectives and the following conclusions were drawn from the result of the data analysis.

5.1.1 The results of test scores analysis

To see the effects of the Polya's Problem-Solving Model, the test scores obtained by the students in pretest and posttest were analyzed using paired sample t-test (Table 4.1). The mean score of pretest and posttest were 6.66 and 9.34 respectively. Pretest was administered before the treatment and posttest was administered after the treatment. It was evident from the result that the comparison of mean of pretest and posttest indicated that there was significant increase in academic achievement with the use of Polya's Problem-Solving Model with 2-tailed significant value 0.000. This accepted the hypothesis ($H^{1)}$ which stated that instruction using Polya's Problem-Solving Model increases the academic achievement of fourth grade students. Thus, the study concluded that as a result of incorporating Polya's problem-solving model it helped the students to perform well in mathematics by scoring significantly higher after the treatment.

5.1.2 The results of time series records

The second purpose of the study was to find out analyzing ability of the students in mathematics problem solving with the help of Polya's Problem-Solving Model. To so do, time series record of every individual subjects were maintained by conducting a short test every end of the lesson. The test paper consisted of 2 questions and it was assessed using Polya's Problem-Solving rubric (Appendix C). Since the Polya's model consisted of four steps, analyzing ability applied by the students in each steps were analyzed using mean. The comparison of mean within the step (Figure 4.3) and the comparison of subtotal mean raw scores of different ability (Figure 4.4) revealed that the students applied more analyzing skills in the first two steps than that of step 3 and 4. However, the overall mean scores of all the steps in each lesson (Figure 4.2) indicated that the analyzing ability of the students did improve with repeated practice of Polya's Problem-Solving Model. This accepted the second hypothesis (H^2) which stated that Polya's Problem-Solving Model will help increase students anlyzing ability. Thus, the conclusion could be made that the Polya's Problem-Solving Model was indeed a very useful means of encouraging students to use their analyzing skills to solve mathematics problem in a very effective way. It was also concluded that with the improvement in analyzing skills, it brought improvement in learning achievement in mathematics.

5.2 Discussion

The overall results of the study revealed that Polya's Problem-Solving Model increased the academic achievement and improved analyzing skills of fourth grade students in mathematics. The model was introduced in the beginning of the treatment to ensure smooth flow of the lesson. However, the researcher did come across some problems in the process of the treatment. The problems faced were:

1. Omission of steps in solving the problems

Since the researcher introduced all the steps right away, it became difficult for the students to follow all the steps while solving the problem. They skipped one or two steps of the Problem-Solving Model. So, in order to keep the students on track, the researcher had to reteach the steps, modeling with different situations. Gradually, with more practice and reminder they started using all the steps and were more likely to understand the problem, find both missing and available information, make plan and successfully completed their math problem. This was evident from the result of the posttest (Figure 4.2).

2. Classroom management

In this study, most of the class activities were group based learning. The students were made to fiddle with blocks and counters to help them solve problems. The researcher noticed that the group was dominated by the high achievers and there wasn't much learning taking place in the low achievers. To overcome the classroom management problem and to equalize the learning process among both groups of learners the researcher sat with each group and acquainted them by conducting short question answer session. The researcher kept in mind never to discourage the high achievers while doing so. During the course it was observed that the students had difficulty in using the methods as it was new to them and they had limited ideas on checking for error. After the observation the researcher provided additional support, feedback and was bought back to their group task.

5.2.1 Pre and post achievement test

Pretest consisted of 20 multiple choice questions carrying one mark each. It was administered before the treatment to examine the knowledge level of the learners in solving mathematics problem. The same questions were used for the posttest at the end of the treatment. The finding of the result showed that the mean score of the pretest was 6.66 (below average) and posttest was 13.78 (above average).

It was apparent from the mean scores of the two that the students had less knowledge and ideas on the process involved in solving the problems systematically before the treatment but the progression in learning achievement and analyzing ability after the treatment indicated that they have improved their performance in problem solving. The finding that Polya's Problem-Solving Model improved the academic achievement was congruent with Ali (2010) who examined the effect of problem solving method on students' achievement in teaching mathematics at elementary level and Eksi (2005) who carried out the study to investigate the effect of different problem-solving strategies. Their finding showed that there was significant gain between the pretest and posttest in teaching students with different problem-solving model. Students performed better when taught with problem-solving model than they did in the conventional method. Kousar Perveen (2010), Erin McCarthy Bowman (2010) and victor Uche (1986) also conducted similar research and they concluded that the Polya's Problem-Solving method resulted in higher achievement level and helped student turn from passive listeners to active information receivers, free self-learner and problem solver. Such instructional model was useful in enabling the students to learn new knowledge by facing the problem to be solved instead of feeling bored. It also shifted the emphasis of educational programs from teaching to learning (child-centered learning).

The gain in academic achievement was supported by constructivist theory. According to constructivist theory, students learn more when they interact and engage actively in learning. In this study most of the activities were done in groups where the students discussed, questioned each other, supported each other and got to the solution. The students were posed with guided questions which were within the framework of Polya's Model and this helped the students to link the information with their finding whereby gaining new set of knowledge. Further it was also supported by behaviorist theory and gestalt theory. According to behaviorist theory, students learn better through trial and error methods. In this study the students were encouraged to try out different methods in solving the problem when their plans or the first method failed. Doing so encouraged the students to further apply the idea on different situation. Gestalt theory on the other hand believes that if students are taught to represent the problem situation they can easily get to the solution. Polya's Problem-Solving Model served as an organizational strategy for the students and helped them to pull out key information needed to solve the problem before they began their explanations (Bowman, 2010).

In this study the students demonstrated that they enjoyed learning the new Problem-Solving Models despite some difficulties that they encountered in the process. The model was found very beneficial especially in identifying the known and unknown information given in the question and the last step (look back) where they had to go back and check for any error though they could not perform well in this step. So far it was always the teacher who checked the whole process and the students did the correction after that. But with the inclusion of Polya's Problem-Solving Model students were encouraged and motivated to apply their own thinking ability, look for any left out information and to come up with their own innovative and creative ways to check the answers. Polya stated that, much can be gained by taking time to reflect and look back at what you have done, what worked and what didn't. This will help predict what strategy to use to solve future problems.

5.2.2 Time series records

The second finding of the study was that the Problem-Solving Model enhanced the analyzing ability of the students. This was proved by the mean scores from the time series record which was maintained to observe the application of analyzing skills in each step (Figure 4.2) which indicated progression in application of analyzing skills with every new lesson. It was noticed that by the end of the treatment students were all able to analyze the question, plan thoughtfully and proceed with solving part, check their answer and check for any error made during the process. Nonetheless, while comparing the mean scores within the steps it demonstrated that students could apply more analyzing skills in step 1(Understand the problem) and step 2 (Devise a plan) of Polya's model. The possible reasons to account for such high analyzing ability could be because of the group activities where the students experienced the power of learning through discussion, helping each other and receiving instant feedbacks from the instructor. Furthermore, the first two steps of the Problem-Solving Model were found easy to apply. In addition to that the researcher also asked questions and inspired the student to explain their understanding of the given question. This helped the researcher to take note of the progression of analyzing skills of every individual student and provided guidance accordingly.

The result of the time series record also revealed that the mean scores of the last two steps (Carry out and Look back) were comparatively low than the first two steps. The reason for the low scores in step 3 (Carry out) could be because some of the students left out few identified information while solving the problem and some students wrongly interpreted the identified information. It could be because of the new methods prescribed in the new mathematics text. Present fourth grade students were dealing with the new text for the first time. Some possible reason to account for low mean scores in the last step (Look back) could be because students were introduced to such methods for the first time. The students never experienced the importance of checking their work. The responsibility was assumed to be the teacher's work. Students had very limited ideas on checking their work. Above all, they fell under concrete operational stage where they could think logically but required concrete concept, as their brains were not well-developed to engage themselves into higher level of thinking (Jean Piaget, 1973). The researcher had help the students by organizing enriching activities and provided blocks and counters to let them get hands on learning experiences with the new methods. Students were also guided in checking their errors by explaining the strategies such as the usage of multiplication tables and repeated addition.

The result of the mean comparison between the different ability groups revealed that the different groups had experienced the same that is they found the first two steps easier than the last two steps. Yet, the level of analyzing skills applied by each group in each steps were different. While comparing the scores obtained under each step which was provided as per the Polya's Problem-Solving rubric, it was noted that the high achiever mostly fell under level three and four of the assessment rubric, average students fell under level three and two. The low achieve fell under level 2.

The reason could be because the high achievers had good language content. They faced less difficulty in understanding and identifying the known and unknown information compared to the low achievers. Average students could also do well in identifying the information. High achiever could use all the information acquired and proceed with plan but the low achiever missed out some information and failed to get to the solution. However, the trend (Figure 4.4) showed that all the groups did apply analyzing skills with the guidance of the Polya's Model despite the difficulties encountered. This was evident from the scores obtained in the test conducted to see the analyzing skills which was above average. Polya;s Model helped the students to analyze the problem and to organize their thinking before they solved the problem (Erin 2010). This was also evident form the posttest result which was significantly higher than the pretest result.

Thus, the final conclusion drawn from the findings were that the Polya's Problem-Solving Model improved the analyzing ability of the students whereby resulting in higher learning achievement in mathematics.

5.3 Recommendations

5.3.1 Recommendations for the study

This study was executed to find the effects of Polya's Problem-Solving Model in enhancing the academic achievement and to see the analyzing skills employed by the student in solving the mathematics problem. Based on the findings the following recommendations have been made and could be evidence for meaningful and better performance in mathematics which indeed is the greatest concern of mathematic teachers of Bhutan.

1. Polya's Problem-Solving Model is very effective instructional tool to acquaint students to solve math problem systematically with better understanding.

2. It is essential to use varieties of learning tools like blocks, counters etc so that the learning becomes active, meaningful and lively.

3. The findings of the study would be helpful for mathematic teachers in Bhutan to improve their teaching strategy.

4. This study will serve as reference for future researcher to carry out studies on similar fields.

5.3.2 Recommendation for further research

Taking in account some of the limitation of the study I would like to make some recommendations for future research as follows:

1. Since the study revealed that the students had difficulty in applying analyzing skill in the last two steps of Polya's model, a research could be conducted using Polya's Model with more emphasis on the last two steps and find out the factors affecting their performance.

2. It would be interesting to conduct a comparative study of Polya's Problem-Solving Model with another problem-solving model and observe the difference in learning achievement and application of analyzing ability.

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APPENDICES

Appendix A

(Lesson Plans)

Lesson plan 1

Learning Unit: Unit 2 Multiplication and Division Facts

Topic: Division

Class: IV

Time: block period(100 mins)

Content: Division: Division is splitting into two equal parts or groups. It is the result of fair sharing. One meaning of division is sharing.

- In a sharing problem you know the total number of items and the number of groups.
- You need to find the number of items each group gets.

Eg: 20 ÷ 5 can mean 20 items shared equally in 5 groups.

Lesson Objective(s): (KPA)By the end of the lesson a child will be able to:

- ✓ Explain division as sharing after teacher's demonstration.(knowledge)
- ✓ Solve sharing problems using different methods (number line, arrays, using blocks with minimum error.(psychomotor)
- ✓ Participate actively and solve division problem with enthusiasm (affective)

Previous knowledge: simple division and multiplication table

Teaching learning materials: work sheet, counters

Steps	Teacher's activity	Students activity
Understand the problem	Teacher will keep students in groups of 4, 5 and 6 Give counters of different numbers to the groups Ask students to share the counters equally among themselves	Students will count the counters They will write down how many of them are going to sharing the counters equally Guess the answer

Devise a plan	Ask student to think of a plan (how to go about sharing the counters) to share the counters among them equally.	In groups they will discuss how to share the counters.
Carry out	Monitor their work Ask guided questions: 1. How many counters did you have? 2. What information did you record? 3. How did you plan to share the counters? 4. Did ever member get same number of counters	Start sharing the counters equally
Look back	Ask question to make sure they have all the information 1. Did you use all the information correctly? 2. Did your plan work? 3. Was there any leftover counters? Why? 4. Can you use the same number of counters to be shared by different no of people? Give example. 5. What could be your definition of Division	Answer the questions Make conclusion that division is sharing firstly in sentence form and then represent the equation.

Exercise- Division as sharing

Group name: Group A

Members: ....

Date: ....

Materials: counters, chart paper, marker,

Marks obtained:.....................

Solve the following questions in groups using the answer sheet provided below. Use any method.

Q1: There are 20 toy cars to be shared by 4 children. How many toy cars will each get?

Q2: Some chocolate are to be shared by equally by 6 students. There are no chocolate left over. How many chocolate might there have been to start with? Find 2 possible answers.

Q1.

Answer sheet

Steps	**Items needed/guided questions**	**Complete**
Step 1. Understand the Problem	a. What is the question asking you to find? b. Look for the information given in the problem.	
Step 2. Make a plan	a. List the methods that can used to solve the problem	
Step 3. Solve the problem	a. Chose the method listed in step 2 and solve the problem.	
Step 4. Look back	a. Check if your step 1-3 are correct b. Compare the answer with your friends. c. Use multiplication table	
Assessment	Group presentation Students will explain their findings	**Scores**

	Teacher will observe the group presentation and assess the work using Polya's Problem-solving rubric.(Appendix C) Teacher will ask questions to check their analyzing skill. Eg. Can same number of toys be shared by 6 children? Why?	

Appendix B

Time Series (Analyzing questions)

Worksheet 1: Division as Grouping

Name: ....

Date: ....

Mark obtained: ....

Solve the following questions.

Q3. 28 people are going on a car trip. Each car holds 4 people. How many cars are needed?

Q4. Karma is reading a book that has 45 pages. He reads 9 pages each night. How many nights will he read to complete the book?

Worksheet 2: Multiplication and Division fact family

Name: ____________________ Date:________

Solve the following questions

Q1. Karma wants to sell 48 chocolate bars to get a prize. If one box contains 6 chocolate bars, how many boxes he need to sell. Show and explain how you are going to use multiplication and division to find out the total number of boxes.

Q2. What fact family does this show?

35-7= 28

28-7= 21

21-7= 14

14-7= 7

7-7= 0

Worksheet 3: Dividing Tens and Hundreds

Name: ______________________________ Date: __________

Solve

Q1. Write the division sentences of the given model. Describe your understanding.

Q2. 1250 books are to be packaged in a bag. If 5 books are to be put in one bag, how many bags are needed?

Worksheet 4: Estimate the quotient

Name: ______________________________ Date: __________

Answer the following questions

Q2. A flock of 278 birds was formed when 4 small flocks flew together. The small flocks were all about the same size.

a. What number close to 278 would you choose to estimate and why?

b. About how many birds were in each small flock?

Q3. Describe a situation where you might wish to estimate 257 ÷ 3. Show your answer.

Appendix C

(Assessment Rubrics on Polya's Problem-Solving Model)

Scores: 4-Excellent, 3-Very good, 2-Good,1-Poor

Components	4	3	2	1
Define and understand the Problem	Shows clear understanding of problem and identifies all the known and unknown information given in the question.	Shows clear understanding of the problem and identifies some known and unknown information.	Shows partially developed understanding of the problem and identifies a few known and unknown information.	Shows limited understanding of the problem.
Devising a plan or strategy to solve the problem	Student can recognize or classify the structure of the problem, consider one or more strategies, and coordinate several processes into a strategy.	Student can begin to think about more than one method of solution. They can identify a plan based upon structural aspects of the problem not just keywords and phrases but not always with accuracy.	Student can identify a viable strategy especially when keywords are provided and plan is straightforward.	Student select a strategy without regard to fit. Student does not have ability to consider new strategies even if theirs is clearly not appropriate.
Carry out or execute the the plan	Student can recognize the need for multiple paths to carry out the plan. They can implement plans with several processes or steps	Student frequently recognizes the need for multiple paths to carry out the plan. They can implement plans with limited number of processes or steps	Student does not demonstrate well developed thought or reasoning in carrying out the plan.	Student demonstrates minimal thought or reasoning in carrying out the plan.
Looking back	Student always analyzes or synthesizes results from a wide range of perspectives	Student frequently analyzes or synthesizes results from more than one perspective.	Student sometime analyzes or synthesizes results.	Student does not analyze or synthesize results.

Source: http://www.hostos.cuny.edu/oaa/pdf/genedf09_problem%20solving%20rubric.pdf

Appendix D

(Pre and post achievement test questions)

Test Blue Print

	Level of cognition			
	Remembering	**Understanding**	**Applying**	**Analyzing**
Division as sharing		Q1		Q18
Division as grouping				Q5
Multiplication and division fact family	Q10			
Dividing tens and hundreds	Q8		Q4	Q16
Estimating quotient		Q 19		

School: Trongsa Primary School Date:

Time: 50 mins

Class: IV

Subject: Mathematics

Marks: 20 marks (1 x 20 = 20 marks)

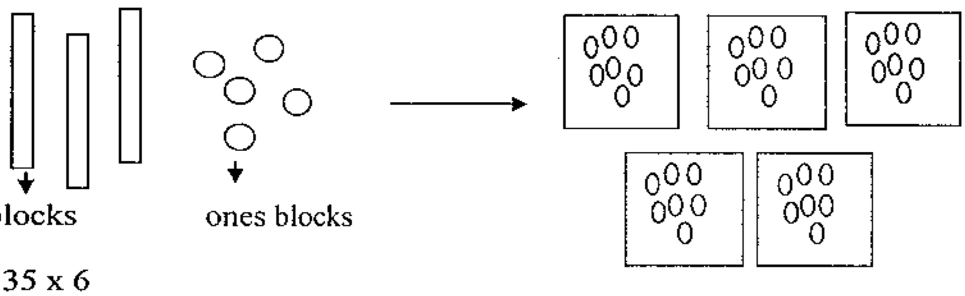
Direction: Answer all the questions.

Read the questions carefully and circle the correct answer.

Q1.54 sweets are shared by 6 friends equally. How many sweets will each get?

a. 9 sweets
b. 8 sweets
c. 7 sweets
d. 6 sweets

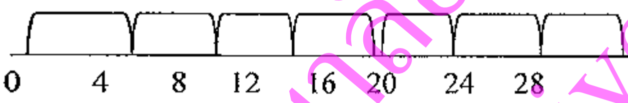
4. What does the given model represent?

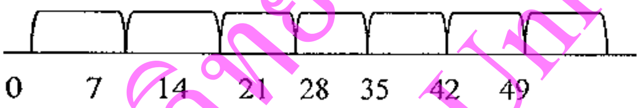


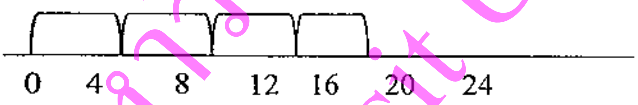
a. 35 x 6
b. 35 ÷ 5
c. 6 x 5
d. 30 ÷ 5

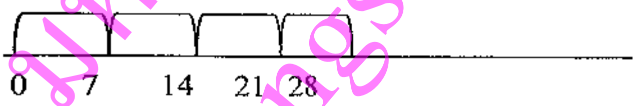
Q5. Which number line matches the division equation given below?

28 ÷ 4

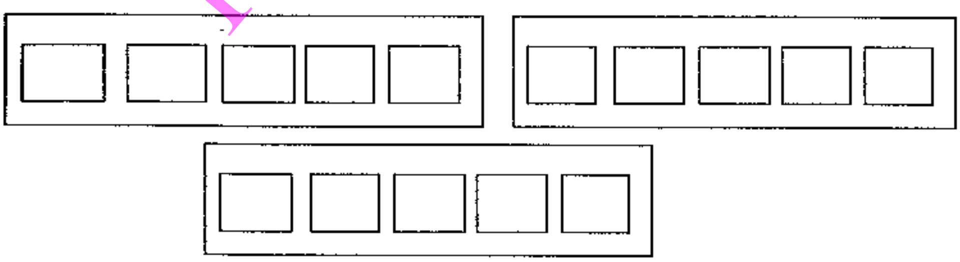
a.


b.


c.


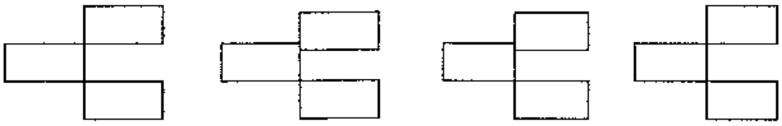
d.


Q8. What is the division sentence of the base ten models shown below?



a. 15000 ÷ 3
b. 150 ÷ 3
c. 1500 ÷ 3
d. 15 ÷ 3

Q10. What fact family does the picture show?



a. 12 ÷ 3= 4, 12 ÷ 4 = 3
b. 12 x 3 = 36, 12 x 4 = 48
c. 4 + 3= 7, 12 - 4 = 8
d. 12 x 3 = 36, 3 +12= 15

Q16. There are 170 shoes in a store. To find out the number of pairs of shoes we:

a. Divide 170 by 2
b. Divide 2 by 170
c. Multiply 170 by 4
d. Multiply 170 by 2

Q18. Pema said you can divide 48 ÷ 8 by subtracting 8s from 48 till you get 0. Then you count the numbers of 8s you subtracted. Pema used all the steps given below EXCEPT:

a. Skip counting
b. Multiples of 8
c. Repeated subtraction
d. Using base ten blocks

Q19. A store clerk packaged 138 pencils in a group of 5. About how many packaged did the clerk make?

a. 27 packaged
b. 30 packaged
c. 20 packaged
d. 31 packaged

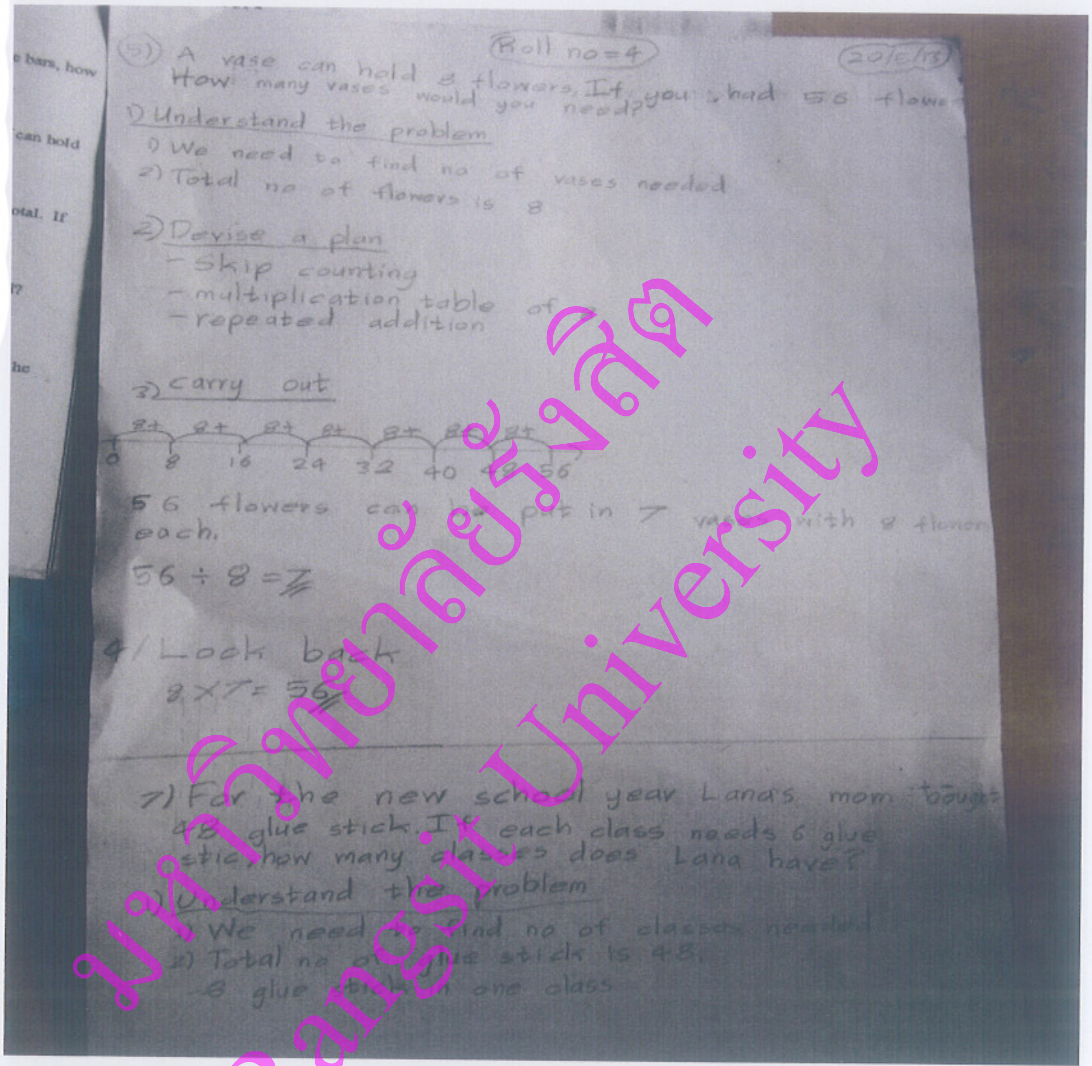
Appendix E

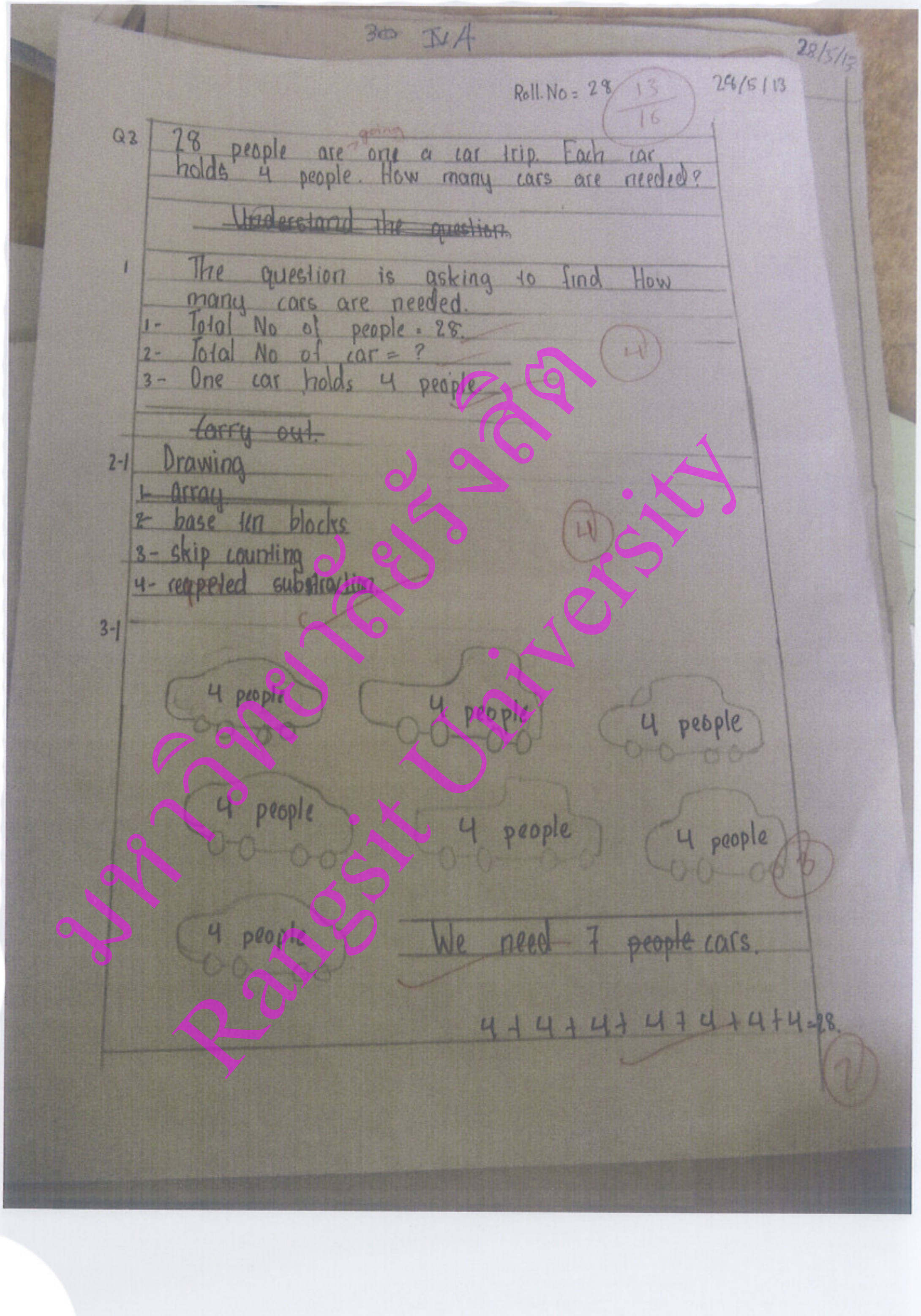
(IOC for the pre and post questionnaires)

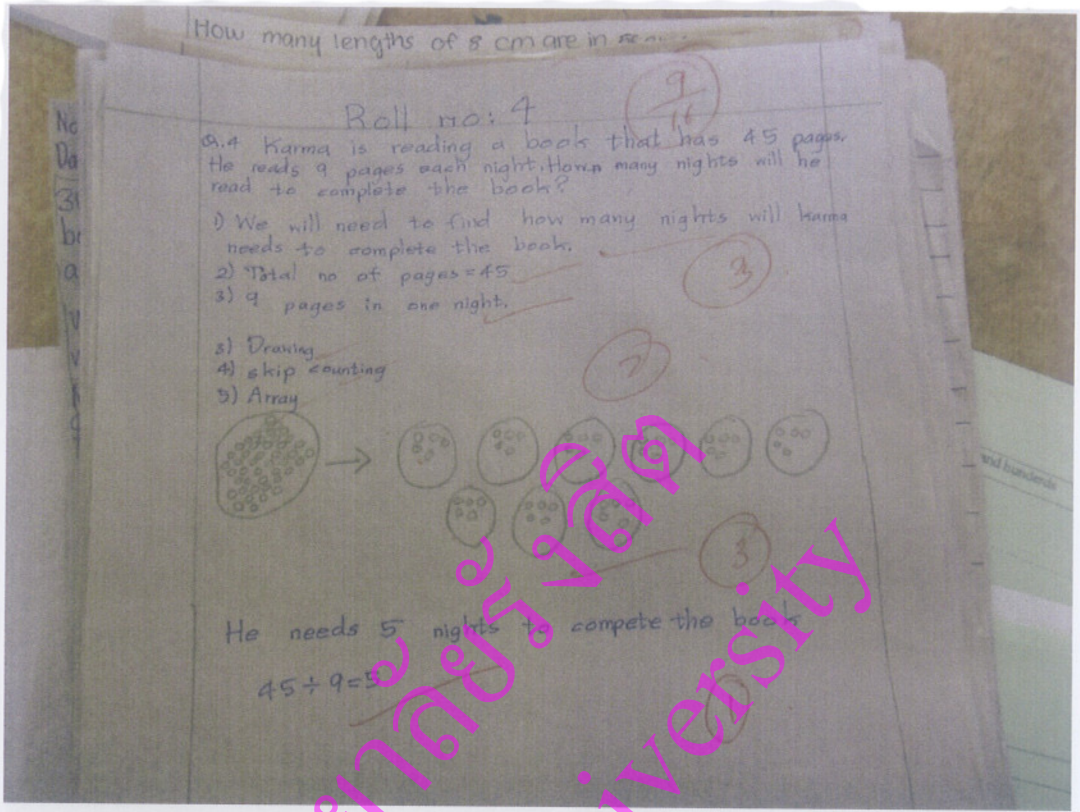
Items	Expert 1	Expert 2	Expert 3	IOC	
	(+1) ,(0) ,(-1)	(+1) ,(0) ,(-1)	(+1) ,(0) ,(-1)		
1	+1	+1	+1	1.00	Accepted
2	+1	+1	+1	1.00	Accepted
3	+1	+1	+1	1.00	Accepted
4	+1	+1	+1	1.00	Accepted
5	+1	+1	+1	1.00	Accepted
6	+1	+1	+1	1.00	Accepted
7	+1	+1	+1	1.00	Accepted
8	+1	+1	+1	1.00	Accepted
9	+1	+1	+1	1.00	Accepted
10	+1	+1	+1	1.00	Accepted
11	+1	+1	+1	1.00	Accepted
12	+1	+1	+1	1.00	Accepted
13	+1	+1	+1	1.00	Accepted
14	+1	+1	+1	1.00	Accepted
15	+1	+1	+1	1.00	Accepted
16	+1	+1	+1	1.00	Accepted
17	+1	+1	+1	1.00	Accepted
18	+1	+1	+1	1.00	Accepted
19	+1	+1	+1	1.00	Accepted
20	+1	+1	+1	1.00	Accepted

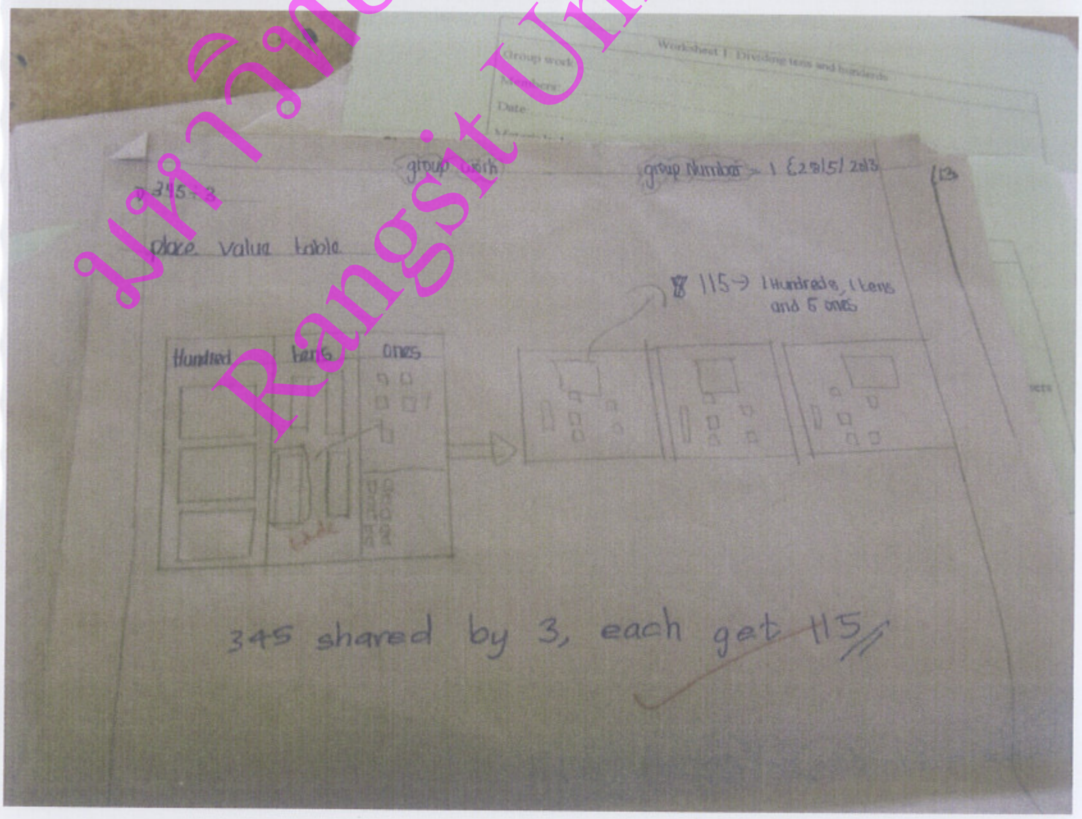
Appendix F

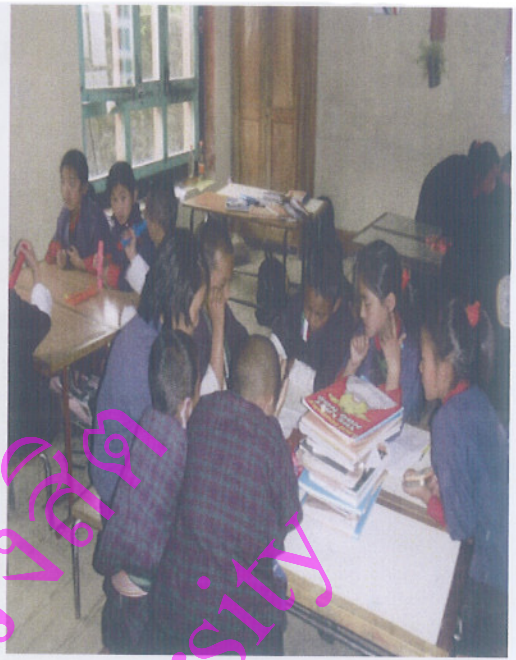
(Sample of students work using Polya's Model)

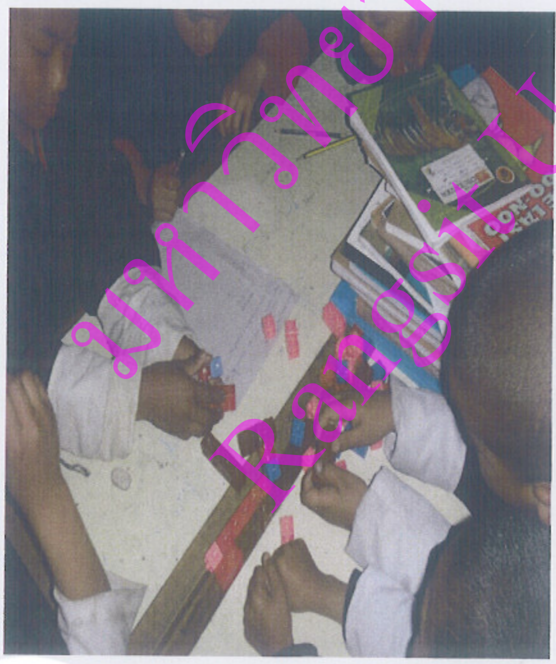



3∞ IVA
28/5/13
Roll. No = 28
13/16
29/5/13
Q2 28 people are going one a car trip. Each car holds 4 people. How many cars are needed?
Understand the question.
1 The question is asking to find How many cars are needed.
1- Total No of people = 28.
2- Total No of car = ?
3- One car holds 4 people
4
Carry out.
2-1 Drawing
1- array
2- base ten blocks
3- skip counting
4- reppeled subtraction
4
3-1
4 people
4 people
4 people
4 people
4 people
4 people
4 people
We need 7 people cars.
4+4+4+4+4+4+4=28.
3
2


How many lengths of 8 cm are in
Roll no: 4
Q.4 Karma is reading a book that has 45 pages. He reads 9 pages each night. How many nights will he read to complete the book?
1) We will need to find how many nights will Karma needs to complete the book.
2) Total no of pages = 45
3) 9 pages in one night.
3) Drawing
4) skip counting
5) Array
He needs 5 nights to compete the book
45÷9=5


Worksheet 1: Dividing tens and hundreds
Group work
Members
Date
group work
group Number - 1
345÷3
place value table
115 → 1 Hundreds, 1 tens and 5 ones
Hundred
tens
ones
345 shared by 3, each get 115





Appendix G

(Approval letter from MoE)

FROM :SHSS TRONGSA FAX NO. :03521285 5 Jun. 2013 12:43PM P1

དཔལ་ལྡན་འབྲུག་གཞུང་། ཤེས་རིག་ལྷན་ཁག།

Royal Government of Bhutan
Ministry of Education
Human Resource Division

Educating for GNH

MoE/HRD-HRDs/INSET/22/2013/ 4355

3rd May 2013

To Whom It May Concern

This is to certify that following five teachers are currently pursuing M.Ed. in Curriculum and Instruction at Rangsit University, Thailand under Trongsa Poenlop Scholarship starting June 2012 for duration of 23 months.

1. *Yangzom(EID#201001565), Sherubling HSS, Trongsa*
2. *Yeshi Dema(EID#200801407), Trongsa PS, Trongsa*
3. *Ugyen Namgay(EID#200801581), Langthel LSS, Trongsa*
4. *Yeshey Nidup (EID#200901686), Bartsham MSS, Trashigang*
5. *Prem Kumar Ghalley(EID#201001216), MSS, S/Jongkhar*

The Royal Civil Service Commission (RCSC) has approved above teachers to carry out their Research Study in Bhutan based on the University's recommendation letter and the Ministry's request made on the Research Study. In addition the Ministry has also noted that the research topics are very relevant to their current job responsibilities.

In view of above, the Ministry of Education would like to request all authority concerned to kindly render necessary support while they collect research information and data to enable them to have a reliable research analysis and conclusion. For any clarification please contact HRD, MoE at 02-335402 during office hours.

The Ministry of Education wishes them best of luck in their endeavor.

(Tshesum Dawa)
Offtg. Chief HRO

HUMAN RESOURCE OFFICER
Ministry of Education
Thimphu : Bhutan

MINISTRY OF EDUCATION ★ THIMPHU: BHUTAN ★

VITAE

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	M.Ed. (Curriculum and Instruction)
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